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B.M.S. COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU – 560004

SEMESTER END EXAMINATION – SEPT/OCT 2023

M.Sc in Mathematics – 4th Semester

ENTIRE AND MEROMORPHIC FUNCTIONS

(NEP Scheme 2021-2022 Onwards)

Course Code: MM405T **QP Code: 14005**

Duration: 3 Hours **Max. Marks: 70**

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. a) Define order and type of an entire function. Find order and type of $f(z) = \cos z$.

   b) If f is a non-constant entire function with $\liminf\limits_{r\to\infty}\frac{M(r,f)}{r^n} = O(1)$. Then show that f is a polynomial of degree at most $'n'$. **(7+7)**

2. a) Suppose f is a transcendental entire function, then prove that $\rho(f) = \rho(f')$.

   b) If $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is an entire function. Let $\mu(r,f)$ and $\nu(r,f)$ be the maximum term and rank of $f(z)$ respectively. Then prove that $\log M(r,f) \sim \log \mu(r,f)$.
   Further show that $\rho = \limsup\limits_{r\to\infty}\frac{\log\log \mu(r,f)}{\log r}$.

   (7+7)

3. a) Define Borel exceptional value and give example. State and prove Borel's theorem for an entire function.

   b) Define asymptotic value and asymptotic path of an entire function. Prove that every Borel exceptional value is an asymptotic value for an entire function. **(7+7)**

4. a) Prove that (i) $T\left(r,\ f \mp \frac{1}{f}\right) = 2T(r,f) + O(1)$.

   (ii) $T(r,Kf) = T(r,f) + O(1)$, K is a non-zero constant.

   b) Suppose $f(z) = \frac{a_p z^p + a_{p-1}z^{p-1} + \cdots + a_0}{b_q z^q + b_{q-1}z^{q-1} + \cdots + b_0}$, $a_p \neq 0, b_q \neq 0$, then prove that

   $T(r,f) = O(\log r)$. **(6+8)**

5. a) Define proximate order. If f is an entire function of finite order ρ $(0 < \rho < \infty)$, then show that $\liminf\limits_{r\to\infty}\frac{\log M(r,f)}{T(r,f)} < \infty$.

   b) State and prove Cartan's lemma. **(5+9)**

6. a) With usual notations, prove that: (i) $0 \leq \delta(a, f) \leq 1$ for all $a \in \mathbb{C}$. (ii) evP $\Rightarrow$ evB.
   b) State and prove Nevanlinna's second fundamental theorem for three small functions.

(4+10)

7. a) If f is a meromorphic function of finite order with $\delta(\alpha, f) = \delta(\beta, f) = 1$ for a complex constants $\alpha, \beta \in \mathbb{C}, \alpha \neq \beta,$ then prove that $T(r, f') \sim 2T(r, f)$.
   b) State and prove Nevanlinna's defect relation for the meromorphic function in terms of deficient values. **(7+7)**

8. a) Suppose that two meromorphic functions $f_1(z)$ and $f_2(z)$ share five distinct values in the complex plane, then show that either $f_1(z) \equiv f_2(z)$ or $f_1(z)$ and $f_2(z)$ are constants.
   b) State and prove Montel's theorem. **(7+7)**

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